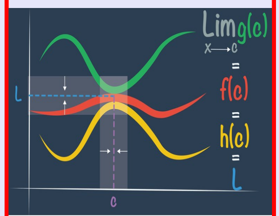
Calculus I

Lecture 17



Given $4x^2 + 9y^2 = 36$

$x = x(t)$, $y = y(t)$

Find $\frac{dy}{dt}$ when $x = -2$, $y = \frac{2\sqrt{5}}{3}$, and $\frac{dx}{dt} = 3$.

$$\frac{d}{dt}\left[4x^2 + 9y^2\right] = \frac{d}{dt}\left[36\right]$$

$$4 \cdot 2x \cdot \frac{dx}{dt} + 9 \cdot 2y \cdot \frac{dy}{dt} = 0$$

$$8(-2) \cdot 3 + \overset{6}{\cancel{18}} \cdot \frac{2\sqrt{5}}{\cancel{3}} \frac{dy}{dt} = 0$$

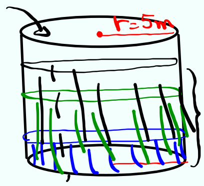
$$-48 + \boxed{12\sqrt{5}} \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{\overset{4}{\cancel{48}}}{\cancel{12}\sqrt{5}} = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

A cylindrical tank has a radius of 5m is being filled with water at the rate of 3 m^3/min.

$\frac{dV}{dt} = 3\ m^3/min.$

How fast its height increasing?



h is increasing, find $\frac{dh}{dt}$

$V = \pi r^2 h$ but $r = 5m$ $V = 25\pi h$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$3 = 25\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{25\pi}$ m/min.

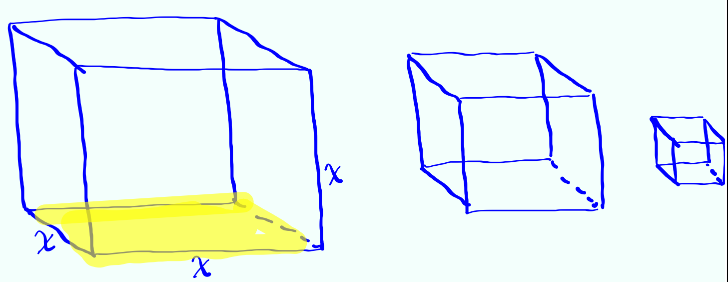
Apr 21-8:56 AM

A cubic piece of ice is melting such that its surface area decreases at 1 cm^2/min.

$\frac{dS}{dt} = -1$

How fast is its edge changing when the edge is 2cm?

$\frac{dx}{dt}$ when $x = 2$



$S = 6x^2$ $\frac{dS}{dt} = 6 \cdot 2x \cdot \frac{dx}{dt}$ $-1 = 6 \cdot 2(2) \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{-1}{24}$ cm/min.

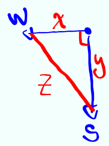
Apr 21-9:03 AM

Two Cars start moving from the Same point.

one goes South @ 60 mi/h.

other one goes west @ 25 mi/h.

At what rate distance between them Changing two hrs later?



$x^2 + y^2 = z^2$ $\quad \frac{dx}{dt} = 25$, $\frac{dy}{dt} = 60$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$ find $\frac{dz}{dt}$ 2 hrs later

$x = 2 \cdot 25 = 50$

$y = 2 \cdot 60 = 120$

$50 \cdot 25 + 120 \cdot 60 = 130 \cdot \frac{dz}{dt}$

$8450 = 130\frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 65 \text{ mi/h}}$

$x^2 + y^2 = z^2$

$50^2 + 120^2 = z^2$

$16900 = z^2$

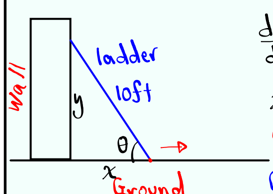
$z = 130$

Apr 21-9:11 AM

A 10-ft ladder leaning againt a wall.

The bottom of the ladder slides away from the wall at the rate of 1 ft/s. $\frac{dx}{dt} = 1$

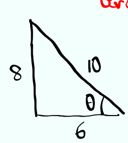
How fast the angle between the ladder and the ground changes when the bottom of the ladder is 6 ft from the wall?



$\frac{d\theta}{dt}$ $\quad x = 6$

$x^2 + y^2 = 10^2$

$6^2 + y^2 = 100 \longrightarrow y = 8$



$\sin\theta = \frac{8}{10}$

$\cos\theta = \frac{x}{10}$

$10\cos\theta = x$

$10 \cdot -\sin\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$

$-\cancel{10} \cdot \frac{8}{\cancel{10}} \cdot \frac{d\theta}{dt} = 1$

$-8\frac{d\theta}{dt} = 1$

$\boxed{\frac{d\theta}{dt} = -\frac{1}{8} \text{ Rad/s}}$

Apr 21-9:21 AM